20/3/17

$\rightarrow$ συνέχεια άσκησης $\quad tr(AB) = tr(BA)$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ & & & \\ a_{n1} & \dots & & a_{nn} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ & & & \\ b_{n1} & \dots & & b_{nn} \end{pmatrix} = \begin{matrix} a_{11}b_{11} + a_{12}b_{21} + \dots + a_{1n}b_{n1} \\ + a_{21}b_{12} + \dots + a_{2n}b_{n2} \\ + \dots \end{matrix}$$

$$tr\left(\begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ & & & \\ b_{n1} & \dots & & b_{nn} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ & & & \\ & & & \\ a_{n1} & \dots & & a_{nn} \end{pmatrix}\right)$$

A όμοιος $B \Rightarrow$ υπάρχει αντιστρέψιμος πίνακας P τέτοιος ώστε

$$A = P^{-1} B P$$

$$tr(A) = tr(P^{-1}BP) = tr(BPP^{-1}) = tr(BI_n) = tr(B)$$

$\phi : V \rightarrow V$

$$tr(\Phi) = tr([\phi]_a^a) = tr([\phi]_b^b)$$

$$[\Phi]_a^a = [I]_b^a [\phi]_b^b [I]_a^b$$

$[\phi]_\alpha^\alpha$, $[\phi]_b^b$ όμοιοι

$$\det(\phi) \underline{\underline{\text{ορισμός}}} \det([\Phi]_\alpha^\alpha) = \det([\phi]_b^b)$$

Όμοιοι πίνακες έχουν την ίδια ορίζουσα

$$A = P^{-1}BP \Rightarrow \det(A) = \det(P^{-1}BP) = \det(B) \det P \det P^{-1}$$

βαθμίδα γραμμικής απεικόνισης

$$T: V \to W$$

$$\operatorname{rank}(T) = \dim \operatorname{Im} T = \operatorname{rank}\left([T]_a^{\beta}\right) = \operatorname{rank}\left([T]_{\gamma}^{\delta}\right)$$

6) $T: \mathbb{R}_n[x] \to \mathbb{R}_n[x]$

ίχνος T
βαθμίδα T
ορίζουσα T

$T(f(x)) = f(x)$

$$[T]_a^a = \begin{array}{ccccc} & & T(x^2) & & \\ \left(\begin{matrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 2 & 0 & \cdots & \vdots \\ 0 & 0 & 0 & 3 & & \vdots \\ \vdots & \vdots & \vdots & \vdots & & n \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{matrix}\right) \\ T(1) & T(x) & & T(x^3) & T(x^n) \end{array}$$

$tr(T) = 0$

$\det(T) = 0$

$\operatorname{rank}(T) = n-1$

7) $\phi: \mathbb{R}^3 \to \mathbb{R}^3$ $\quad \phi((x,y,z)) = (y+z, x+z, x+y)$

$$[\phi]_a^a = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

$tr(\phi) =$ (αθροίζω τα στοιχεία της διαγωνίου) $= 0$

$\det(\phi) = 1+1 = 2$

$\operatorname{rank}(\phi) = 3$

$a = \{(1,0,0), (0,1,0), (0,0,1)\}$

8) $T: \mathbb{F}^{n\times 1} \to \mathbb{F}^{n\times 1}$

$T(b) = Ab$

$[T]_{\mathcal{E}}^{\mathcal{E}} = A$

rank $(T) =$ rank(A)

det $(T) =$ det(A)

tr $(T) =$ tr(A)

9) $\psi: \mathbb{R}^3 \to \mathbb{R}^2$ $\quad \psi(x,y,z) = (x, 4y+6z)$

$\phi: V \to \mathbb{R}^2$ $\quad \phi = \psi/_V$

$V = \{(a,b,c) \in \mathbb{R}^3 / a+2b+3c=0\}$

rank ψ

rank ϕ

rank(ψ)

$$[\psi]_{\mathcal{E}}^{e} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 6 \end{pmatrix}$$

Άρα rank$\psi = 2$

$\mathcal{E} = \{(1,0,0),(0,1,0),(0,0,1)\}$

$e = \{(1,0),(0,1)\}$

$= \{(-2s-3t, s, t) / s,t \in \mathbb{R}\}$

$(1 \ 2 \ 3 | 0)$

$a = 2s - 3t$

$b = s$

$c = t$

$= \{(s(-2,1,0), t(-3,0,1)\}$

$= \langle(-2,1,0), (-3,0,1)\rangle$

$\phi(-2,1,0)=(-2,4)$

$\phi(-3,0,1)=(-3,6)$

$$[\phi]_a^{\mathcal{E}} = \begin{pmatrix} -2 & -3 \\ 4 & 6 \end{pmatrix}$$

$a=\{(-2,1,0),\ (-3,0,1)\}$ βάση του V

Φυλλάδιο #2 | Άσκ. 3

Ιδιοτιμές και ιδιοχώρους

$$A=\begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 3 \end{pmatrix}$$

$$\chi_A(x)=\det(A-xI_3)=\begin{vmatrix} 2-x & 1 & 1 \\ 2 & 3-x & 2 \\ 1 & 1 & 2-x \end{vmatrix} = \begin{vmatrix} 2-x & 0 & 1 \\ 2 & 1-x & 2 \\ 1 & -1+x & 2-x \end{vmatrix}$$

$\Sigma_2-\Sigma_3$

$$= (1-x)\begin{vmatrix} 2-x & 0 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 2-x \end{vmatrix} = (1-x)\left((2-x)^2-2-1+2(2-x)\right)$$

$$=(1-x)(x^2+4-4x-3+4-2x)$$

$$=(1-x)(x^2-6x+5)=(1-x)(x-5)(x-$$

Ιδιοτιμές 1 (διπλή)
5 (απλή)

$V(1) = \{ X \in \mathbb{R}^{3\times 1} \mid (A - 1 \cdot I_3) X = 0_{3\times 1} \}$

$$(A - I_3 | 0) = \left(\begin{array}{ccc|c} 2-1 & 1 & 1 & 0 \\ 2 & 3-1 & 2 & 0 \\ 1 & 1 & 2-1 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad x+y+z=0$$

$x = s - t$
$y = s$
$z = t$

$$\left\{ \begin{pmatrix} -s-t \\ s \\ t \end{pmatrix} \,\middle|\, s,t \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$V(3) = \{ X \in \mathbb{R}^{3\times 1} / A - 5I) X = 0 \}$

$$\left(\begin{array}{ccc|c} 2-5 & 1 & 1 & 0 \\ 2 & 3-5 & 2 & 0 \\ 1 & 1 & 2-5 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} -3 & 1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 1 & 1 & -3 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 2 & -2 & 2 & 0 \\ -3 & 1 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & 4 & -8 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x + y - 3z = 0$
$y - 2z = 0$
$z = t$

$x = t$
$y = 2t$
$z = t$

$$= \left\langle \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \right\rangle$$

$$B = \begin{pmatrix} 0 & i & i \\ i & 0 & i \\ i & i & 0 \end{pmatrix}$$

$$\chi_B(x) = \begin{vmatrix} -x & i & i \\ i & -x & i \\ i & i & -x \end{vmatrix} \xrightarrow{r_3 \to r_1 + r_2 + r_3} = \begin{vmatrix} -x & i & i \\ i & -x & i \\ 2i-x & 2i & 2i-x \end{vmatrix} =$$

$$= (2i - x) \begin{vmatrix} -x & i & i \\ i & -x & i \\ 1 & 1 & 1 \end{vmatrix} \xrightarrow{r_1 \to r_1 - ir_3} = (2i - x) \begin{vmatrix} -x-i & 0 & 0 \\ 0 & -x-i & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (2i - x)(x+i)^2$$

$-i$ (διπλή)

$2i$ (απλή)

$$V(2i) = \{ X \in \mathbb{C}^{3\times 1} \mid (B - 2i\, I) X = 0 \}$$

$$\left(\begin{array}{ccc|c} -2i & i & i & 0 \\ -i & -2i & i & 0 \\ i & i & -2i & 0 \end{array}\right) \begin{array}{l} r_1/i \\ r_2/i \\ r_3/i \end{array} \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ -2 & 1 & 1 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x + y - z = 0 \qquad x = t$$
$$y - z = 0 \qquad y = t$$
$$z = t$$

$$V(2i) = \left\{ \begin{pmatrix} t \\ t \\ t \end{pmatrix} \mid t \in \mathbb{C} \right\} = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

4) $A = \begin{pmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix}$

$$\chi_A(x) = \begin{vmatrix} 3-x & 0 & -1 \\ 0 & 2-x & 0 \\ -1 & 0 & 3-x \end{vmatrix} = (2-x)(-1)^{2+2} \begin{vmatrix} 3-x & -1 \\ -1 & 3-x \end{vmatrix} =$$

$$(2-x)\left((3-x)^2 - 1\right) = (2-x)\left((3-x-1)(3-x+1)\right) = (2-x)(2-x)(3-x$$

Ιδιοτιμές 2 με αλγεβρική πολλαπλότητα 2
και 4 με αλγ. πολ/τητα 1

$$V(2) = \{ X \in \mathbb{R}^{3\times 1} \mid (A - 2I)X = O_{3\times 1} \}$$

$$(A - 2I \mid 0) = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x - z = 0$ $\qquad$ $x = s$
$y = t$
$z = s$

$$V(2) = \left\{ \begin{pmatrix} s \\ t \\ s \end{pmatrix} \middle| s, t \in \mathbb{R} \right\} = \left\{ s\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \middle| s, t \in \mathbb{R} \right\}$$

$$= \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$

$V(4) = \{ X \in \mathbb{R}^{3\times 1} \,/\, (A-4I)X = 0 \}$

$$(A-4I \,|\, 0_{}) = \left(\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 \\ -1 & 0 & -1 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x + z = 0$ $\qquad$ $x = -t$

$y = 0$

$z = t$

$$V(4) = \left\langle \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

Για κάθε ιδιοτιμή η γεωμ. πολ/τητα ισούται με την αλγεβρ. άρα ο A είναι διαγωνίσιμος

$$[f_A]_B^B = [I]_\varepsilon^B \, [f_A]_\varepsilon^\varepsilon \, [I]_B^\varepsilon$$

$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$